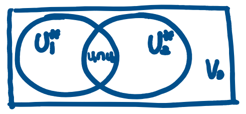
期中考试突击

2019年4月15日 17:06

予定 Fri. 4:00～6:00 PM



$\ker T\oplus V_0$

$(U_1\cap U_2)\oplus U_1^*\oplus U_2^*\oplus V_0 \xrightarrow{T} W$

vector space V → W

有限维先想基

$\dim V<\infty$ → $\exists$ a basis

指[illegible]～同构

$\dim V=\infty$ → $\nexists$ a finite basis.

$\{v_i\}$ basis

spans V → linear combination $\sum_{i=1}^{n} a_i v_i$

linearly indep.

contradictory

$\mathbb{F}^\infty \neq \operatorname{span}\{(1,0,\cdots),(0,1,\cdots)\cdots\}$

$T\in\boxed{\mathcal{L}(V,W)}$ → vector space

$\ker T\subset V \leftarrow \rightarrow \operatorname{Im}T\subset W$

$\ker T$: $\{0\}$ only → $\hookrightarrow$

$\operatorname{Im}T$: "=" → $\twoheadrightarrow$

$$\boxed{\dim\ker T+\dim\operatorname{Im}T=\dim V}$$

W → $\operatorname{Im}T\oplus W_0$

$V\times W$ $(\dim V+\dim W, <\infty)$

$(v_1,0),$
$(v_2,0),$
$\vdots$
$(v_n,0),$ → $\dim V$

$(0,w_1),$
$\vdots$
$(0,w_m).$ → $\dim W$

张老子一起（附加题）！

题目会[illegible]考核.

$U\subset V$

$V/U:=\{\bar v\}=\{v+U\mid v\in V\}$

$(\dim V-\dim U)$

$$\boxed{\begin{array}{l}\pi:\ V\longrightarrow V/U \\ \quad\ v\longmapsto \bar v \qquad (\hookrightarrow,\ \twoheadrightarrow,\ 同构) \\ +\ \text{FTLA}\end{array}}$$

典范映射（同构）

$V/\ker T\cong\operatorname{Im}T$

$$\boxed{\begin{array}{l}\tilde T:\ V/\ker T\longrightarrow W \\ \quad u+\ker T\longmapsto Tv\end{array}}$$

同构.

$$\begin{tikzcd}
 & & & & & & \operatorname{coim}\varphi \arrow[d, equal] & \\
0 \arrow[r, hook, "i"] & \ker\varphi \arrow[r, hook, "i"] & V \arrow[r, "\pi"] \arrow[rrr, bend right, "\varphi"'] & V/\ker\varphi \arrow[r, "\bar\varphi", "\cong"'] & \operatorname{Im}\varphi \arrow[r, hook, "i"] & W \arrow[r, "\theta"] & W/\operatorname{Im}\varphi \arrow[r] & 0 \arrow[lllllll, bend left]
\end{tikzcd}$$

有可能会出题

$V':=\mathcal{L}(V,\mathbb{F})\cong V$

basis of V $v_i \longleftrightarrow v_i'$ basis of V'

$v_i'(v_j)=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}$

Boolean Type Construction

annihilator

$V'\supset U^0=\{\varphi\in V'\mid\forall u\in U,\ \varphi(u)=0\}$

$$\boxed{\dim U+\dim U^0=\dim V}\ !$$

$i\ :\ i(u)=u$

$i'\ :\ V'\longrightarrow U'$

!

$V'':=\mathcal{L}(V',\mathbb{F})$

$v''(f)=f(v)$

$$\boxed{(v,f)=(f,v)=f(v)=v(f)}$$

$$\boxed{\Lambda:\ (\Lambda v)(\varphi)=\varphi(v).}$$