Mechanics Summary

Log Creative

1 只涉及三维坐标的质点运动学

自然坐标系

$$x(t) - x_0 = \int_0^t \left(v_0 + \int_0^t a dt \right) dt \tag{1.1}$$

$$\boldsymbol{r}(t) - \boldsymbol{r}_0 = \int_0^t \left(\boldsymbol{v}_0 + \int_0^t \boldsymbol{a} dt \right) dt \tag{1.2}$$

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r} \tag{1.3}$$

$$\boldsymbol{a} = \boldsymbol{a}_t + \boldsymbol{a}_n = \frac{dv}{dt}\boldsymbol{e}_t + \frac{v^2}{\rho}\boldsymbol{e}_n \tag{1.4}$$

极坐标系

$$\dot{\boldsymbol{e}_{\boldsymbol{r}}} = \dot{\theta}\boldsymbol{e}_{\boldsymbol{\theta}} \tag{1.5}$$

$$\dot{\boldsymbol{e}_{\boldsymbol{\theta}}} = -\dot{\theta}\boldsymbol{e}_{\boldsymbol{r}} \tag{1.6}$$

$$\boldsymbol{v} = \dot{r}\boldsymbol{e}_r + r\dot{\theta}\boldsymbol{e}_\theta \tag{1.7}$$

$$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2 \right) \boldsymbol{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} \right) \boldsymbol{e}_\theta \tag{1.8}$$

相对运动(伽利略变换)

$$\boldsymbol{v} = \boldsymbol{v}' + \boldsymbol{v}_f \tag{1.9}$$

$$\boldsymbol{a} = \boldsymbol{a}' + \boldsymbol{a}_f \tag{1.10}$$

匀速转动

$$\boldsymbol{v} = \boldsymbol{v}' + \boldsymbol{\omega} \times \boldsymbol{r} \tag{1.11}$$

$$\boldsymbol{a} = \boldsymbol{a}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}') + 2\boldsymbol{\omega} \times \boldsymbol{v}' \tag{1.12}$$

2 加入了力的牛顿运动定律

牛顿第二定律

$$\boldsymbol{F} = m\boldsymbol{a} \tag{2.1}$$

虚拟力

$$\boldsymbol{F}_i = m\boldsymbol{a'} - m\boldsymbol{a} \tag{2.2}$$

科里奥利力

$$ma' = ma - m\omega \times (\omega \times r') + 2mv' \times \omega \tag{2.3}$$

3 更广泛适用的动量

质点系动量定理

$$\boldsymbol{F}_{ex} = \frac{d\boldsymbol{p}}{dt} \tag{3.1}$$

质心

$$\boldsymbol{F}_{ex} = m\boldsymbol{a}_c \tag{3.2}$$

$$\boldsymbol{r}_C = \frac{\int \boldsymbol{r} dm}{\int dm} \tag{3.3}$$

4 具有守恒性质的衡量 功与能

引入定义

$$\boldsymbol{F} = m\boldsymbol{C} \tag{4.1}$$

$$\boldsymbol{F} = -\boldsymbol{\nabla} U \tag{4.2}$$

$$\boldsymbol{C} = -\boldsymbol{\nabla} \Psi \tag{4.3}$$

$$\boldsymbol{A} \cdot d\boldsymbol{A} = AdA \tag{4.4}$$

质点系中功能原理

$$W_{ex} + W_{ic} + W_{in} = E_k(b) - E_k(a) \tag{4.5}$$

$$W_{ex} + W_{in} = E(b) - E(a) \tag{4.6}$$

$$E_k = \frac{1}{2}mv_C^2 + E_{kC} = \frac{1}{2}mv_C^2 + \sum_i \frac{1}{2}m_i v_i'^2 \tag{4.7}$$

$$W'_{ex} + W'_{in} = E' - E'_0 \tag{4.8}$$

碰撞

$$e = \frac{v_2 - v_1}{u_1 - u_2} \in [0, 1] \tag{4.9}$$

$$v_1 = \frac{m_1 - em_2}{m_1 + m_2}u_1 + \frac{(1+e)m_2}{m_1 + m_2}u_2 \tag{4.10}$$

$$v_2 = \frac{(1+e)m_1}{m_1 + m_2}u_1 - \frac{em_1 - m_2}{m_1 + m_2}u_2 \tag{4.11}$$

$$\Delta E = \frac{1}{2}(1 - e^2)\frac{m_1 m_2}{m_1 + m_2}(u_1 + u_2)^2 \tag{4.12}$$

5 力矩的积分 角动量

力矩

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F} \tag{5.1}$$

力偶

$$|\boldsymbol{M}_{\text{duality force}}| = Fd \tag{5.2}$$

角动量

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} \tag{5.3}$$

角动量定理

$$\boldsymbol{M}_{ex} = \frac{dL}{dt} \tag{5.4}$$

6 质点力学的组合 刚体力学

角量

$$\varphi(t) - \varphi_0 = \int_0^t \left(\omega_0 + \int_0^t \alpha dt \right) dt \tag{6.1}$$

$$\boldsymbol{a} = \boldsymbol{a}_t + \boldsymbol{a}_n = \frac{d\boldsymbol{\omega}}{dt} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \tag{6.2}$$

转动惯量

$$J = \int r^2 dm \tag{6.3}$$

$$M = J\alpha \tag{6.4}$$

$$L = J\omega \tag{6.5}$$

刚体	转动惯量
圆环	mR^2
圆柱	$\frac{1}{2}mR^2$
圆筒	$\frac{1}{2}m(R_1^2 + R_2^2)$
细棒	(中部)$\frac{1}{12}ml^2$
圆球	$\frac{2}{5}mR^2$
薄球壳	$\frac{2}{3}mR^2$

平行轴定理、正交轴定理

$$J_A = J_C + md^2 \tag{6.6}$$

$$J_z = J_x + J_y \tag{6.7}$$

刚体的动能定理

$$W_{ex} = \int M_z d\varphi = \frac{1}{2}J\omega^2 - \frac{1}{2}J\omega_0^2 \tag{6.8}$$

平面平行运动
动能

$$E_k = \frac{1}{2}J\omega^2 \tag{6.9}$$

纯滚动

$$v_C = R\omega \tag{6.10}$$

$$a_C = R\alpha \tag{6.11}$$

进动

$$\boldsymbol{M} = \boldsymbol{\Omega} \times \boldsymbol{L} \tag{6.12}$$

7 周期性的运动 振动

简谐振动

$$m\ddot{x} = -kx \tag{7.1}$$

$$x = A\cos(\omega t + \varphi) \tag{7.2}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{7.3}$$

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} \tag{7.4}$$

$$tan\varphi = \frac{-v_0}{\omega x_0} \tag{7.5}$$

$$E = \frac{1}{2}kA^2 \tag{7.6}$$

谐振子

$$\ddot{x} + \omega^2 x = 0 \tag{7.7}$$

振动的合成

平行、同频率

$$\begin{cases} x_1 = A_1\cos(\omega t + \varphi_1) \\ x_2 = A_2\cos(\omega t + \varphi_2) \end{cases} \tag{7.8}$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)} \tag{7.9}$$

$$\varphi = \tan^{-1}\frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} \tag{7.10}$$

平行、近频率

$$\begin{cases} x_1 = A\cos(\omega_1 t + \varphi_1) \\ x_2 = A\cos(\omega_2 t + \varphi_2) \end{cases} \tag{7.11}$$

$$x = x_1 + x_2 = 2A\cos\frac{\omega_1 - \omega_2}{2}t\cos\left(\frac{\omega_1 + \omega_2}{2} + \varphi\right) \tag{7.12}$$

$$\Delta\nu = \left|\frac{\omega_1 - \omega_2}{2\pi}\right| \tag{7.13}$$

垂直、同频率

$$\begin{cases} x = A_x\cos(\omega t + \varphi_x) \\ y = A_y\cos(\omega t + \varphi_y) \end{cases} \tag{7.14}$$

$$\frac{x^2}{A_x^2} + \frac{y^2}{A_y^2} - \frac{2xy}{A_xA_y}\cos(\varphi_x - \varphi_y) = \sin^2(\varphi_x - \varphi_y) \tag{7.15}$$

垂直、不同频率

李萨如图形，注意角度起始坐标轴。

8 超越实体的波

简谐波

$$y(x,t) = A\cos\left[\omega\left(t - \frac{x}{u}\right) + \varphi\right] \tag{8.1}$$

$$y(x,t) = A\cos\left(\omega t - kx + \varphi\right) \tag{8.2}$$

$$y(x,t) = A\cos\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \varphi\right] \tag{8.3}$$

波动方程

$$u = \frac{\lambda}{T} = \frac{\omega}{k} \tag{8.4}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 y}{\partial x^2} \tag{8.5}$$

$$u_\parallel = \sqrt{\frac{E}{\rho}} \tag{8.6}$$

$$u_\perp = \sqrt{\frac{G}{\rho}} \tag{8.7}$$

$$\frac{\partial^2 y}{\partial t^2} = u^2\frac{\partial^2 y}{\partial x^2} \tag{8.8}$$

$$u = \sqrt{\frac{F_T}{\rho_l}} \tag{8.9}$$

波的能量与强度

$$\Delta E_k = \frac{1}{2}\rho\Delta V\left(\frac{\partial y}{\partial t}\right)^2 \tag{8.10}$$

$$\Delta E_p = \frac{1}{2}E\Delta V\left(\frac{\partial y}{\partial x}\right)^2 \tag{8.11}$$

$$\Delta E = \rho\Delta V\omega^2A^2\sin^2\omega\left(t-\frac{x}{u}\right) \tag{8.12}$$

$$\varepsilon = \frac{\Delta E}{\Delta V} = \rho\omega^2A^2\sin^2\omega\left(t-\frac{x}{u}\right) \tag{8.13}$$

$$\boldsymbol{I} = \frac{1}{2}\rho\omega^2A^2\boldsymbol{u} \tag{8.14}$$

球面波

$$A \propto \frac{1}{r} \tag{8.15}$$

干涉

$$y_1 = A_1\cos(\omega t+\varphi_1-kr_1) \tag{8.16}$$

$$y_2 = A_2\cos(\omega t+\varphi_2-kr_2) \tag{8.17}$$

$$\Delta = \varphi_1-\varphi_2+k(r_2-r_1) \tag{8.18}$$

$\Delta = 2n\pi$ 加强；$\Delta = (2n+1)\pi$ 减弱。

驻波

$$y_1 = A\cos(\omega t-kx+\varphi_1) \tag{8.19}$$

$$y_2 = A\cos(\omega t+kx+\varphi_2) \tag{8.20}$$

$$y = y_1+y_2 = 2A\cos\left(kx+\frac{\varphi_2-\varphi_1}{2}\right)\cos\left(\omega t+\frac{\varphi_2+\varphi_1}{2}\right) \tag{8.21}$$

$kx+\frac{\varphi_2-\varphi_1}{2}=n\pi$ 波腹； $kx+\frac{\varphi_2-\varphi_1}{2}=\frac{2n+1}{2}\pi$ 波节。

简正模式

$$\nu_n = \frac{n}{2l}\sqrt{\frac{F_T}{\rho_l}} \tag{8.22}$$

多普勒效应

$$\nu_R = \frac{u+v_R}{u-v_S}\nu_S \tag{8.23}$$

$$\nu' = \frac{1+\frac{v\cos\theta}{u}}{1-\frac{v\cos\theta}{u}}\nu = \left(1+\frac{2v\cos\theta}{u}\right)\nu(\text{if}\quad v\ll u) \tag{8.24}$$

(考虑相对论)

$$\nu = \sqrt{\frac{c+v}{c-v}}\nu' \tag{8.25}$$

9 光速不变的相对论

$$\beta = \sqrt{1 - \frac{v^2}{c^2}} \tag{9.1}$$

尺缩、钟慢

$$l = l_0 \beta \tag{9.2}$$

$$t = \frac{t_0}{\beta} \tag{9.3}$$

洛伦兹变换

(正变换)

$$\begin{cases} x' = \frac{x - vt}{\beta} \\ y' = y \\ z' = z \\ t' = \frac{t - \frac{vx}{c^2}}{\beta} \end{cases} \tag{9.4}$$

(逆变换)

$$\begin{cases} x = \frac{x' + vt'}{\beta} \\ y = y' \\ z = z' \\ t = \frac{t' + \frac{vx'}{c^2}}{\beta} \end{cases} \tag{9.5}$$

洛伦兹速度变换

(正变换)

$$\begin{cases} u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}} \\ u'_y = \frac{u_y \beta}{1 - \frac{vu_x}{c^2}} \\ u'_z = \frac{u_z \beta}{1 - \frac{vu_x}{c^2}} \end{cases} \tag{9.6}$$

(逆变换)

$$\begin{cases} u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}} \\ u_y = \frac{u'_y \beta}{1 + \frac{vu'_x}{c^2}} \\ u_z = \frac{u'_z \beta}{1 + \frac{vu'_x}{c^2}} \end{cases} \tag{9.7}$$

动量与能量

$$m = \frac{m_0}{\beta} \tag{9.8}$$

$$E = mc^2 \tag{9.9}$$

$$\boldsymbol{p} = m\boldsymbol{v} \tag{9.10}$$

$$E^2 = p^2c^2 + m_0^2c^4 \tag{9.11}$$

光子

$$E = pc \tag{9.12}$$