光学突击

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Lecture 1

光波的表示与性质

1.Maxwell 方程组

$$\begin{cases} \nabla \cdot \vec{D} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t} \end{cases}$$

$$\nabla^2 \vec{E} - \mu_0 \varepsilon \frac{\partial^2 \vec{E}}{\partial^2 t} = 0$$

介质中的光速

$$v = \frac{1}{\sqrt{\mu\varepsilon}}, \quad n = \frac{c}{v} = \sqrt{\varepsilon_r}$$

2.平面波解

$$E = E_0 \exp\Phi = E_0 \exp i\left(\vec{k} \cdot \vec{r} - \omega t\right) \qquad v = \nu\lambda = {}^{\lambda}/_{\mathrm{T}} = \frac{\omega}{k}$$

3.波前函数

	平面波	球面波（发散与会聚采用$\rho \ll z$）	柱面波
立体	$\tilde{U}(x,y,z) = E_0 e^{ik(\cos\alpha \cdot x + \cos\beta \cdot y + \cos\gamma \cdot z)}$	$U = \frac{E_0}{r} e^{i\left(\vec{k}\cdot\vec{r} - \omega t\right)}$ 发散 $U = \frac{E_0}{r} e^{ik\sqrt{(x-x_1)^2 + (y-y_1)^2 + z^2}}$ 会聚 $U = \frac{E_0}{r} e^{-ik\sqrt{(x-x_1)^2 + (y-y_1)^2 + z^2}}$	$U = \frac{E_0}{\sqrt{r}} e^{i\left(\vec{k}\cdot\vec{r} - \omega t\right)}$
平面	$\tilde{U}(x,y) = E_0 e^{ik(\cos\alpha \cdot x + \cos\beta \cdot y)}$	$U = \frac{E_0}{z_1} e^{ikz_1} e^{\frac{ik}{2z_1}\left[(x-x_1)^2 + (y-y_1)^2\right]}$ 发散 $U = \frac{E_0}{z_1} e^{ikz_1} e^{\frac{ik}{2z_1}\left[(x-x_1)^2 + (y-y_1)^2\right]}$ 会聚 $U = \frac{E_0}{z_1} e^{-ikz_1} e^{-\frac{ik}{2z_1}\left[(x-x_1)^2 + (y-y_1)^2\right]}$	

$\leftarrow$

$$\rho^2 \ll z\lambda \quad \text{（远场条件）}$$

4.宏观速度

相速度	群速度
等位相面的传输速度	多色光合成波包的传输速度
$v_p=\frac{dz}{dt}=\frac{\omega}{k}=\frac{\frac{2\pi c}{\lambda}}{\frac{2\pi n_p}{\lambda}}=\frac{c}{n_p}$	$v_g=\frac{z}{t}=\frac{d\omega}{dk}=\frac{c}{n+\omega\frac{dn}{d\omega}}=\frac{c}{n-\lambda_0\frac{dn}{d\lambda_0}}$

Lecture 2

光在介质中的传输

5.吸收

$$\tilde{n}=n+\mathrm{i}\boldsymbol{\kappa}$$

$$I=|\boldsymbol{E}|^2=\left|E_0\mathrm{e}^{\mathrm{i}(\boldsymbol{k}z-\omega t)}\right|^2=\left|E_0\mathrm{e}^{\mathrm{i}\left(\frac{\omega}{v}z-\omega t\right)}\right|^2=\left|E_0\mathrm{e}^{\mathrm{i}\left(\frac{\frac{\omega}{c}}{\tilde{n}}z-\omega t\right)}\right|^2=I_0\mathrm{e}^{-\frac{\boldsymbol{2\kappa\omega}}{\boldsymbol{c}}z}$$

吸收因子$\alpha=2\frac{\kappa\omega}{c}=\frac{4\pi k}{\lambda}$

折射率

$$n=\sqrt{\varepsilon_r\mu_r}=\sqrt{\varepsilon_r(1+\chi_{\mathrm{m}})}\approx\sqrt{\varepsilon_r}$$

单一本征频率情形

$$\varepsilon_r=\varepsilon_1+\mathrm{i}\varepsilon_2$$
$$\varepsilon_1=n^2-\kappa^2$$
$$\varepsilon_2=2n\kappa$$

$$n=\frac{1}{\sqrt{2}}\sqrt{\sqrt{\varepsilon_1^2+\varepsilon_2^2}+\varepsilon_1}$$

$$\kappa=\frac{1}{\sqrt{2}}\sqrt{\sqrt{\varepsilon_1^2+\varepsilon_2^2}-\varepsilon_1}$$

柯西公式

$$n(\lambda)=A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}+\cdots$$

Lecture 3

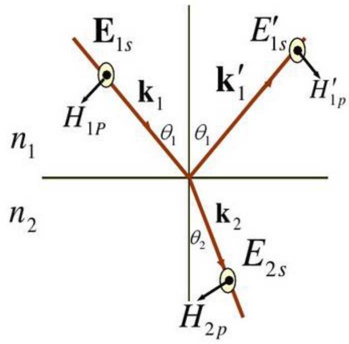
反射&折射（透射）

6.Snell's Law

$$\theta_1' = \theta_1$$
$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$

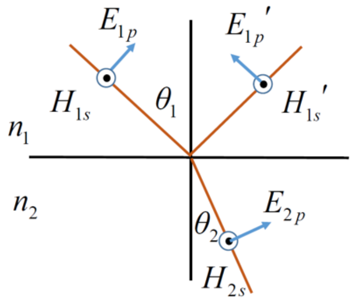
(对角的邂逅)

7.Fresnel's Formula



E 为 s 态、H 为 p 态

$$r_s = \frac{E'_{1s}}{E_{1s}} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} = \frac{\sin(\theta_2-\theta_1)}{\sin(\theta_1+\theta_2)}$$
$$t_s = \frac{E_{2s}}{E_{1s}} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2} = \frac{2\cos\theta_1\sin\theta_2}{\sin(\theta_1+\theta_2)}$$



E 为 p 态、H 为 s 态

$$r_p = \frac{E'_{1p}}{E_{1p}} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2} = \frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}$$
$$t_p = \frac{E_{2p}}{E_{1p}} = \frac{2n_2\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2} = \frac{2\cos\theta_1\sin\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)}$$

方位角

$$\tan\alpha = \frac{E_s}{E_p}$$
$$\tan\alpha_1' = \frac{E'_{1s}}{E'_{1p}} = -\frac{\cos(i_2-i_1)}{\cos(i_2+i_1)}\tan\alpha_1$$
$$\tan\alpha_2 = \frac{E_{2s}}{E_{2p}} = \frac{n_2\cos i_1 + n_1\cos i_2}{n_1\cos i_1 + n_2\cos i_2}\tan\alpha_1$$

<u>**正入射**</u>

$$\boldsymbol{r_s = \frac{n_1-n_2}{n_2+n_1} = -r_p}$$
$$\boldsymbol{t_s = \frac{2n_1}{n_2+n_1} = t_p}$$

8.反射率与透过率

	振幅	光强	光功率($\mathcal{R}_{p,s}+\mathcal{T}_{p,s}=1$)	*线偏振光
反射率	r_s, r_p	$R_P = r_p^2,\ R_S = r_s^2$	$\mathcal{R}_p = R_p, \mathcal{R}_s = R_s$	$\mathcal{R} = \mathcal{R}_s\sin^2\alpha + \mathcal{R}_p\cos^2\alpha$
透射率	t_s, t_p	$T_p = \frac{n_2}{n_1}t_p^2, T_s = \frac{n_2}{n_1}t_s^2$	$\mathcal{T}_p = \frac{\cos\theta_2}{\cos\theta_1}T_p, \mathcal{T}_s = \frac{\cos\theta_2}{\cos\theta_1}T_s$	$\mathcal{T} = \mathcal{T}_s\sin^2\alpha + \mathcal{T}_p\cos^2\alpha$

9.特殊角

9.1 Brewster Angle

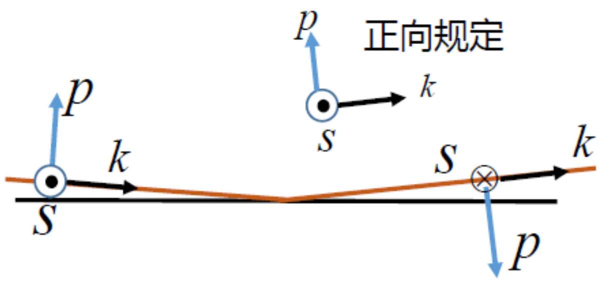
$$\tan\theta_b = \frac{n_2}{n_1}$$

9.2 全反射临界角

$$\sin\theta_c = \frac{n_2}{n_1}$$

10.半波损失

- 光疏 →光密，正入射和掠入射**均有**半波损失
- 光密 →光疏，正入射无半波损失
- 任何情况下，掠入射**有**半波损失



<掠入射>

Lecture 4

干涉

11.干涉条件

$$E(P,t) = E_1(P,t) + E_2(P,t)$$
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta\cos\delta\text{，}\ \delta = k_1 r_1 - k_2 r_2 - (\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)$$

衬比度

$$\gamma = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{\sqrt{m}}{m+1}(1 + \cos\alpha),\ \ I_2 = mI_1$$

12.分波前干涉

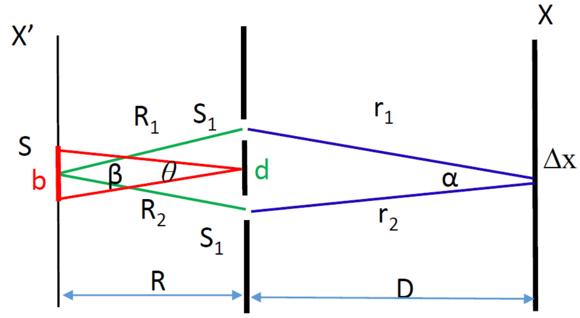
$$\Delta x = \frac{\lambda}{\sin\theta_1 + \sin\theta_2},\ \ k = \frac{1}{\Delta x}$$
$$\Delta x = \frac{\lambda D}{d}$$

空间相干性

光源临界宽度$b_{max} = \frac{\lambda}{\beta}$

横向相干长度$d_{max} = \frac{\lambda}{\theta}$

条纹间距$\Delta x = \frac{\lambda}{\alpha}$

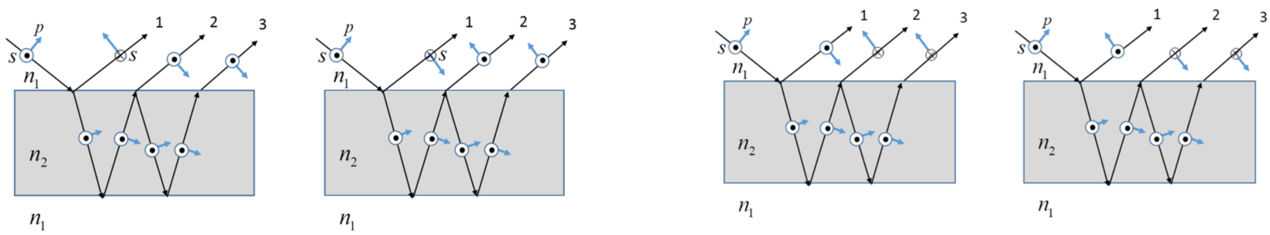


$$\boldsymbol{\lambda = b_{max} \cdot \beta} = d_{max} \cdot \theta = \Delta x \cdot \alpha$$

时间相干性

$$\tau_0 \cdot \Delta\nu = 1, \ L_0 \cdot \frac{\Delta\lambda}{\lambda} = \lambda$$

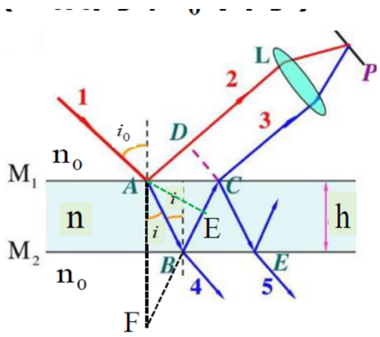
$n_1 < n_2$ $\quad$ $n_1 > n_2$



$\theta_1 < \theta_b$ $\quad$ $\theta_1 > \theta_b$ $\quad$ $\theta_1 < \theta_b$ $\quad$ $\theta_b < \theta_1 < \theta_c$

13.分振幅干涉

13.1 等倾干涉



$$\Delta = 2nh\cos i + \frac{\lambda}{2} = N\lambda, \ \ \delta = \frac{2\pi}{\lambda}\Delta$$

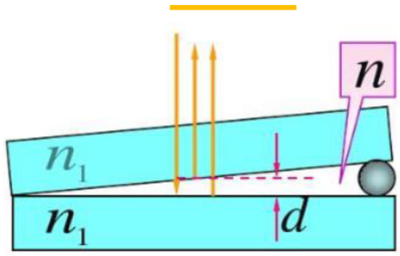
条纹间距

$$e_N = \frac{f}{2}\sqrt{\frac{n\lambda}{mh}}$$

13.2 Michelson

环越往外越密

13.3 等厚干涉

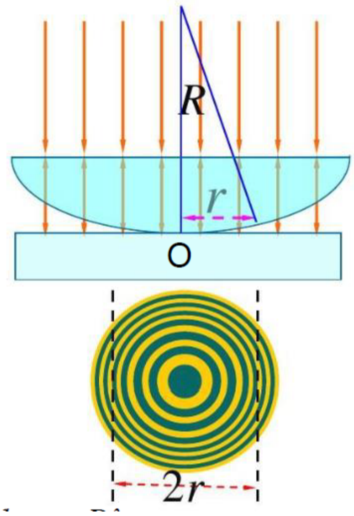


$$\Delta = 2n\text{h} + \frac{\lambda}{2}$$

条纹间距

$$\text{h} = \frac{\lambda}{2n\theta}$$

13.4 牛顿环



$$\Delta = 2n\text{h} + \frac{\lambda}{2}$$

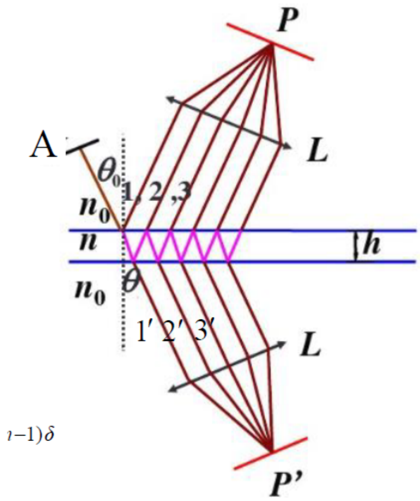
几何关系推半径，要用近似等（忽略高阶小量）。

环越往外越密

14.多光束干涉

精细度系数

$$F = \frac{4R}{(1-R)^2}$$



单界面反射率

$$R = \left(\frac{n-n_0}{n+n_0}\right)^2$$

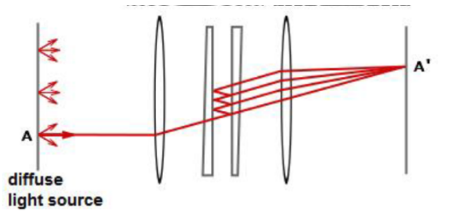
反射强度（思路：等比数列）

$$I_R = \frac{F\sin^2\frac{\delta}{2}}{1+F\sin^2\frac{\delta}{2}} I_0,\ \gamma = 1$$

$$I_T = \frac{1}{1+F\sin^2\frac{\delta}{2}} I_0,\ \gamma = \frac{F}{2+F} = \frac{2R}{1+R^2}$$

法布里-玻罗

锐度



$$\Delta\theta = \frac{\lambda}{2\pi n h \sin\theta}\frac{1-R}{\sqrt{R}}$$

，越小越锐

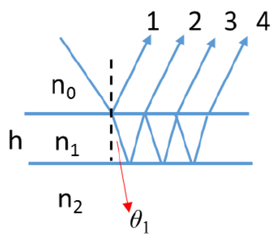
$$\Delta\lambda = \frac{\lambda}{m\pi}\frac{1-R}{\sqrt{R}}$$

分辨本领

$$R_c = \frac{\lambda}{\Delta\lambda}$$

15.薄膜干涉

反射率



$$R = \frac{(n_0-n_2)^2\cos^2\frac{\delta}{2} + \left(\frac{n_0 n_2}{n_1} - n_1\right)^2 \sin^2\frac{\delta}{2}}{(n_0+n_2)^2\cos^2\frac{\delta}{2} + \left(\frac{n_0 n_2}{n_1} + n_1\right)^2 \sin^2\frac{\delta}{2}}$$

- $\sin\frac{\delta}{2} = 0,\ \ R = R_0$

既不增透也不增反

- $\cos\frac{\delta}{2} = 0$

$$R = \left(\frac{n_0 n_2 - n_1^2}{n_0 n_2 + n_1^2}\right)^2,\ \ R = 0, n_1 = \sqrt{n_0 n_2}$$

- 当 n1=n0 或 n1=n2 时，相当于不镀膜
- 当 n0<n1<n2 时，R<R0，具有增透作用
- 当 n0<n1>n2 时，R>R0，具有增反作用

附加 利用等效界面和等效折射率的概念可将多层膜转换成单层膜处理

$n_e = \frac{n_1^2}{n_2}$

$$R_{2N} = \left(\frac{n_a - n_g\left(\frac{n_H}{n_L}\right)^{2N}}{n_a + n_g\left(\frac{n_H}{n_L}\right)^{2N}}\right)^2 = \left(\frac{\frac{n_a}{n_g} - \left(\frac{n_H}{n_L}\right)^{2N}}{\frac{n_a}{n_g} + \left(\frac{n_H}{n_L}\right)^{2N}}\right)^2 \Rightarrow 1$$

Lecture 5
衍射

衍射引论

傍轴条件：

倾斜因子$\frac{1}{2}(\cos\theta_0+\cos\theta)\approx 1$

球面次波函数$\frac{1}{r}\mathrm{e}^{ikr}\approx\frac{1}{r_0}\mathrm{e}^{ikr}$

$$\tilde{U}(P)=\frac{-\mathrm{i}}{\lambda r_0}\iint_{(\Sigma_0)}\widetilde{U_0}(Q)\boldsymbol{e}^{\boldsymbol{ikr}}dS$$

菲涅尔衍射	单缝夫琅禾费衍射	圆孔夫琅禾费衍射	一维矩孔的夫琅禾费衍射	多缝夫琅禾费衍射
$\frac{1}{R}+\frac{1}{b}=k\frac{\lambda}{\rho^2}$	$I(\theta)=I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$	$I(\theta)=I_0\left(\frac{2\mathrm{J}_1(x)}{x}\right)^2$	$I(\theta)=i_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2$	$I(\theta)$ $=i_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$
$\rho_1=\sqrt{\frac{Rb\lambda}{R+b}}$	半角宽度 $\Delta\theta_0=\frac{\lambda}{a}$	$x=\frac{2\pi a\sin\theta}{\lambda}$ $=\frac{\pi D\sin\theta}{\lambda}$ Airy Disk $I_0=\left(\frac{\pi a^2}{\lambda f}\right)^2A^2$	$\alpha=\frac{\pi a\sin\theta}{\lambda}$ $\beta=\frac{\pi b\sin\theta}{\lambda}$	主极大与光栅方程 $\beta=m\pi\Rightarrow\sin\theta=\frac{m\lambda}{d}$
$\rho_k=\sqrt{k}\rho_1$		$x_0=1.22\pi\Rightarrow\Delta\theta_0$ $\approx\frac{1.22\lambda}{D}$	$\Delta\theta_k=\frac{\lambda}{D\cos\theta_k}$	

矢量图解?

Rayleigh criterion(可分辨)

$$\delta\theta>\Delta\theta_0\approx\frac{d}{f}$$

望远镜角放大率

$$M=\frac{f_0}{f_e}$$

最小分辨角

$$\delta\theta_\mathrm{m}\approx\frac{1.22\lambda}{D_o}$$

有效放大率

$$M_\mathrm{eff}=\frac{\delta\theta_\mathrm{e}}{\delta\theta_\mathrm{m}}=\frac{D_o}{D_e}$$

显微镜可分辨的最小线度

$$\delta y_\mathrm{m}\approx 0.61\frac{\lambda_0}{n_0\sin u_0}=0.61\frac{\lambda_0}{\mathrm{N.A.}}$$

$\mathrm{N.A.}$ numerical aperture

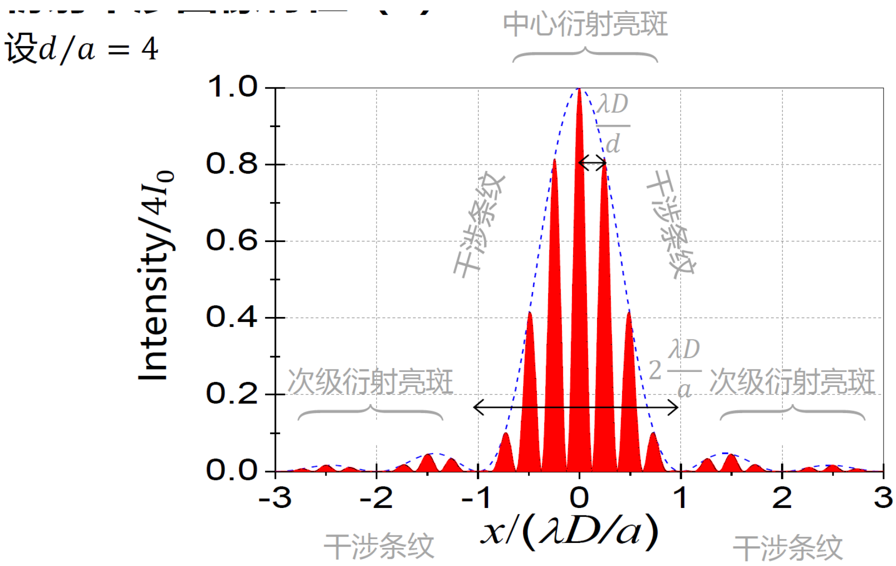
$$D_\theta=\frac{\delta\theta}{\delta\lambda}=\frac{k}{d\cos\theta_k}$$

衍射巴比涅原理(Babinet Principle)

透光率互补

$$\tilde{\boldsymbol{U}}_{\boldsymbol{a}}(P)+\tilde{\boldsymbol{U}}_{\boldsymbol{b}}(P)=\tilde{\boldsymbol{U}}_{\boldsymbol{0}}(P)$$

半波带法



$\frac{\lambda}{n}$片波晶片厚度

$$d_m=\frac{\lambda}{n\Delta n}$$

$$\Delta\theta_k=\frac{\lambda}{Nd\cos\theta_k}$$