Thermal Physics Sheet

Log Creative

1 压强

状态方程

$$pV = \nu RT \tag{1.1}$$

微观公式

$$p = nkT = \frac{2}{3}n\bar{\varepsilon} = \frac{2}{3}n\left(\frac{1}{2}m\bar{v^2}\right) = \frac{2}{3}n\left(\frac{3}{2}m\bar{v_x^2}\right) \tag{1.2}$$

范德瓦尔斯

$$\left(p + \nu^2\frac{a}{V_m^2}\right)(V - \nu b) = \nu RT \tag{1.3}$$

$$b = 4N_A\frac{4}{3}\pi\left(\frac{d}{2}\right)^2 \tag{1.4}$$

Boltzmann

$$p = p_0 e^{-\frac{Mgz}{RT}}(\text{Const. } T) \tag{1.5}$$

$$n = n_0 e^{-\frac{\varepsilon_0}{kT}}(\text{Const. } T) \tag{1.6}$$

2 分量

方均根(速率最大)

$$\sqrt{\bar{v^2}} = \sqrt{\frac{3RT}{M}} \tag{2.1}$$

$$\sqrt{\bar{v_x^2}} = \sqrt{\frac{RT}{M}} \tag{2.2}$$

均值(速率中等)

$$\bar{v} = \sqrt{\frac{8RT}{\pi M}} \tag{2.3}$$

$$\bar{v_x} = \sqrt{\frac{RT}{2\pi M}} \tag{2.4}$$

最可几速率(最概然速率,速率最小)

$$v_p = \sqrt{\frac{2RT}{M}} \tag{2.5}$$

(分量最概然速度大小为0.)

Maxwell

$$\frac{\mathrm{d}N_v}{N} = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} 4\pi v^2 \mathrm{d}v = f(v)\mathrm{d}v \tag{2.6}$$

$$\frac{\mathrm{d}N_{v_x}}{N} = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-\frac{mv_x^2}{2kT}} \mathrm{d}v = f(v_x)\mathrm{d}v \tag{2.7}$$

碰撞

$$N = \frac{1}{4}n\bar{v} \tag{2.8}$$

$$\Delta N_{0\sim\beta v_{p(x)}} = \frac{N}{2}\mathrm{erf}(\beta) \tag{2.9}$$

$$\Delta N_{0\sim\beta v_p} = N\left[\mathrm{erf}(\beta) - \frac{2}{\sqrt{\pi}}\beta e^{-\beta^2}\right] \tag{2.10}$$

$$\mathrm{erf}(\beta) = \frac{2}{\sqrt{\pi}}\int_0^\beta e^{-x^2}\mathrm{d}x \tag{2.11}$$

能量按自由度均分定理

$$\frac{1}{2}m\bar{v^2} = \frac{3}{2}kT \tag{2.12}$$

$$\frac{1}{2}m\bar{v_x^2} = \frac{1}{2}kT \tag{2.13}$$

3 多项

摩尔平均总能量(平动、转动、振动)

$$\begin{aligned}\bar{\varepsilon}_\mathrm{m} &= \frac{1}{2}(t + r + 2s)RT \\ &= \frac{3}{2}RT(\text{single}) \\ &= \frac{5}{2}RT(\text{hard dual}) \\ &= \frac{7}{2}RT(\text{elastic dual}) \\ &= \frac{6}{2}RT(\text{multi})\end{aligned} \tag{3.1}$$

热容

$$C = mc \tag{3.2}$$

$$C_\mathrm{m} = Mc \tag{3.3}$$

$$C_{V,\mathrm{m}} = \frac{1}{2}(t + r + 2s)R \tag{3.4}$$

扩散

$$\sigma = \pi d^2 \tag{3.5}$$

$$\bar{Z} = \sqrt{2}\sigma\bar{v}n \tag{3.6}$$

$$\bar{\lambda} = \frac{1}{\sqrt{2}\sigma n} \tag{3.7}$$

分子按自由程的分布：N_0中自由程大于x的分子数，求微分得连续分布

$$N = N_0 e^{-\frac{x}{\lambda}} \tag{3.8}$$

粘滞、导热、扩散系数

$$\mathrm{d}y = -c\left(\frac{\mathrm{d}x}{\mathrm{d}z}\right)_{z_0}\mathrm{d}S\mathrm{d}t \tag{3.9}$$

$$\eta = \frac{1}{3}\rho\bar{v}\bar{\lambda} \tag{3.10}$$

$$\kappa = \eta c \tag{3.11}$$

$$D = \frac{\eta}{\rho} \tag{3.12}$$

$$c = \frac{C_{V,\mathrm{m}}\nu}{m} \tag{3.13}$$

4 热力学定律

第一定律 加进一个的系统中的热量+对系统所做的功=系统内能的增加

$$Q + A = \Delta U \tag{4.1}$$

第二定律 不可能有这样一个过程，它的唯一结果**只是**从一个热库取出热量，并把它转化为功.

没有任何一台热机，在从T_1取得热量Q_1,而在T_2放出热量Q_2的过程中所做的功比可逆机更大.对于可逆机，

$$W = Q_1 - Q_2 = Q_1\left(1 - \frac{T_2}{T_1}\right) \tag{4.2}$$

系统的熵 如果ΔQ是可逆地加在温度为T的系统中的热量，那么这个系统的熵增为

$$\Delta S = \frac{\Delta Q}{T} \tag{4.3}$$

熵为：

$$S(\boldsymbol{V}, \boldsymbol{T}) = R\left(\ln\boldsymbol{V} + \frac{1}{\gamma - 1}\ln\boldsymbol{T}\right) + a \tag{4.4}$$

当$T = 0$时，$S = 0$(**第三定律**).此时的熵定义为：

在**可逆变化**中，系统所有部分(包括热库)的总熵**不变**.

在**不可逆变化**中，系统的总熵始终不断**增加**.**无摩擦的准静态过程**是可逆的.

通用公式

$$Q_V = \Delta U = \nu C_{V,\mathrm{m}}\Delta T \tag{4.5}$$

$$Q_p = \Delta H = \nu C_{p,\mathrm{m}}\Delta T \tag{4.6}$$

$$A = -\int_{V_1}^{V_2} p\mathrm{d}V \tag{4.7}$$

$$H = U + pV \tag{4.8}$$

$$T\mathrm{d}S = \mathrm{d}U + p\mathrm{d}V \tag{4.9}$$

理想气体

$$\Delta U = \nu C_{V,\mathrm{m}}\Delta T \tag{4.10}$$

$$C_{p,\mathrm{m}} - C_{V,\mathrm{m}} = R \tag{4.11}$$

$$\Delta H = \nu C_{p,\mathrm{m}}\Delta T \tag{4.12}$$

范德瓦尔斯气体

$$\Delta U = \nu\left[C_{V,\mathrm{m}}\Delta T - a\Delta\left(\frac{1}{V_\mathrm{m}}\right)\right] \tag{4.13}$$

$$C_{p,\mathrm{m}} - C_{V,\mathrm{m}} = \frac{R}{1 - \frac{2a(V_\mathrm{m} - b)^2}{RTV_\mathrm{m}^3}} \tag{4.14}$$

$$\Delta H_\mathrm{m} = (C_{V,m} + R)\Delta T - a\Delta\left(\frac{1}{V_\mathrm{m}}\right) + \frac{RT_2 b}{V_2 - b} - \frac{RT_1 b}{V_1 - b} \tag{4.15}$$

5 热力学过程

A:外界对系统所做的功
Q:系统从外界吸收的热量
C_m:摩尔热容
ΔS:熵变
(理想气体)

等容过程 $V=$Const.

$$A = 0 \tag{5.1}$$
$$Q = \nu C_{V,\mathrm{m}}(T_2 - T_1) \tag{5.2}$$
$$C_{V,\mathrm{m}} = \frac{R}{\gamma - 1} \tag{5.3}$$
$$\Delta S = \nu C_{V,\mathrm{m}} \ln \frac{T_2}{T_1} \tag{5.4}$$

等压过程 $p=$Const.

$$A = -p(V_2 - V_1) = -\nu R(T_2 - T_1) \tag{5.5}$$
$$Q = \nu C_{p,\mathrm{m}}(T_2 - T_1) \tag{5.6}$$
$$C_{V,\mathrm{m}} = \frac{\gamma R}{\gamma - 1} \tag{5.7}$$
$$\Delta S = \nu C_{p,\mathrm{m}} \ln \frac{T_2}{T_1} \tag{5.8}$$

等温过程 $T=$Const.

$$A = -p_1 V_1 \ln \frac{V_2}{V_1} = -\nu R T_1 \ln \frac{V_2}{V_1} \tag{5.9}$$
$$Q = -A \tag{5.10}$$
$$C_\mathrm{m} = \infty \tag{5.11}$$
$$\Delta S = \nu R \ln \frac{V_2}{V_1} \tag{5.12}$$

绝热过程 $Q=0$

泊松方程

$$pV^\gamma = \text{Const.} \tag{5.13}$$
$$TV^{\gamma-1} = \text{Const.} \tag{5.14}$$
$$\frac{p^{\gamma-1}}{T^\gamma} = \text{Const.} \tag{5.15}$$

$$A = \frac{p_1 V_1}{\gamma - 1}\left[\left(\frac{V_1}{V_2}\right)^{\gamma-1} - 1\right] = \nu C_{V,\mathrm{m}}(T_2 - T_1) \tag{5.16}$$
$$Q = 0 \tag{5.17}$$
$$C_m = 0 \tag{5.18}$$
$$\Delta S = 0 \tag{5.19}$$

多方过程 $pV^n=$Const.

$$A = \frac{p_1 V_1}{n - 1}\left[\left(\frac{V_1}{V_2}\right)^{n-1} - 1\right] = \frac{\nu R}{n-1}(T_2 - T_1) \tag{5.20}$$
$$Q = \nu\left(C_{V,\mathrm{m}} - \frac{R}{n-1}\right)(T_2 - T_1) \tag{5.21}$$
$$C_m = \frac{\gamma - n}{1 - n} C_{V,\mathrm{m}} \tag{5.22}$$
$$\Delta S = \nu C_{V,\mathrm{m}}(\gamma - n) \ln \frac{V_2}{V_1} \tag{5.23}$$

自由膨胀 $A=0$

这一过程不是准静态过程.

绝热节流 $H=$Const.

焦汤系数

$$\alpha \equiv \lim_{\Delta p \to 0}\left(\frac{\Delta T}{\Delta p}\right)_H = \left(\frac{\partial T}{\partial p}\right)_H \tag{5.24}$$

α	类型(室温下)	效应
+	氮、氧、空气	制冷效应、正效应
−	氢气	制温效应、负效应
0	理想气体	零效应*

* 非理想气体的对应温度为**转换温度**.上转换温度$T^\circ = \frac{2a}{Rb}$.

6 其他热力学方程

效率、冷却系数

$$\eta = \frac{A}{Q_1} = 1 - \frac{|Q_2|}{Q_1} \tag{6.1}$$
$$\varepsilon = \frac{Q_2}{Q_1 - Q_2} \tag{6.2}$$

卡诺、可逆

$$\eta = 1 - \frac{T_2}{T_1} \tag{6.3}$$
$$\varepsilon = \frac{T_2}{T_1 - T_2} \tag{6.4}$$
$$\oint_{\text{invertible cycle}} \frac{\mathrm{d}Q}{T} = 0 \tag{6.5}$$

热力学定律常用(以及 4.9)

$$\left(\frac{\partial U}{\partial V}\right)_V = T\left(\frac{\partial p}{\partial T}\right)_V - p \tag{6.6}$$
$$\mathrm{d}U = \left(\frac{\partial U}{\partial T}\right)_V \mathrm{d}T + \left(\frac{\partial U}{\partial V}\right)_T \mathrm{d}U \tag{6.7}$$

熵

$$W = \frac{N!}{\Pi_{i=1}^n N_i!} \tag{6.8}$$
$$S = k \ln W \tag{6.9}$$

大气温度梯度

$$\frac{\mathrm{d}T}{\mathrm{d}z} = -\frac{\gamma - 1}{\gamma}\frac{T}{p}\rho g \tag{6.10}$$
$$\mathrm{d}p = -\frac{Mgp}{RT}\mathrm{d}z(\text{Boltzmann}) \tag{6.11}$$
$$h = \frac{C_{p,\mathrm{m}} T_0}{Mg}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{6.12}$$

7 相变

克拉珀龙方程衍生式

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{l}{T(v_2 - v_1)} = \frac{u_2 - u_1 + p(v_2 - v_1)}{T(v_2 - v_1)} \approx \frac{l}{Tv_2} \approx \frac{\Delta p}{\Delta T} \tag{7.1}$$

理想气体蒸气压方程

$$\ln p = -\frac{L}{RT} + \text{Const.} \tag{7.2}$$

范德瓦尔斯相变临界点参量

$$\begin{cases} T_k = \dfrac{8a}{27bR} \\ V_{\mathrm{m}k} = 3b \\ p_k = \dfrac{a}{27b^2} \end{cases} \tag{7.3}$$

$$\frac{RT_k}{p_k V_{\mathrm{m}k}} = \frac{8}{3} \tag{7.4}$$

8 常数

$$R = \frac{k}{N_A} \tag{8.1}$$

	Value
R	8.314
k	1.381×10^{-23}
N_A	6.022×10^{23}

$$n = \frac{N_A}{V_\mathrm{m}} \tag{8.2}$$

	Value
V_m(STP)	22.41

$$A = eU \tag{8.3}$$

	Value
e	1.602×10^{-19}C
eV	1.602×10^{-19}J