复习

2022 年 5 月 23 日

1 光线追踪

光线追踪基本原理和算法

光线追踪加速结构

KD-Tree (K-dimensional Tree) 每次切分都是按照某一轴切分，是一个二叉树。

1. 如果中间节点 (internal node) 与光线相交，则和两个子节点都可能相交，子节点继续判断
2. 如果叶子节点 (leaf node) 与光线相交，则叶子节点里的所有物体都要和光线做相交判断。

non-overlapping

BVH (Bouding Volume Hierarchy) may be overlapping

1. 如果光线和叶子节点相交，就和其中所有物体算一遍求交
2. 如果光线和中间节点相交，则分别处理两个子节点，返回更近的一次相交

BRDF (Bidirectional Reflectance Distribution Function)

$$f_r(\omega_i \to w_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} = \frac{dL_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i} \tag{1}$$

$$L_r(\mathbf{p}, \omega_r) = \int_{H^2} f_r(\mathbf{p}, \omega_i \to \omega_r) L_i(\mathbf{p}, \omega_i) \cos\theta_i d\omega_i \tag{2}$$

渲染方程

$$\underbrace{L_o(\mathbf{p}, \omega_o)}_{\text{outgoing radiance}} = \underbrace{L_e(\mathbf{p}, \omega_o)}_{\text{emission}} + \int_{\Omega^+} \underbrace{f_r(\mathbf{p}, \omega_i \to \omega_o)}_{\text{BRDF}} \underbrace{L_i(\mathbf{p}, \omega_i)}_{\text{incident radiance}} \cos\theta_i d\omega_i \tag{3}$$

$$\underbrace{L_o(\mathbf{p}, \omega_o)}_{\text{outgoing radiance}} = \int_{\Omega^+} \underbrace{f_r(\mathbf{p}, \omega_i \to \omega_o) \cos\theta_i}_{\text{(cosine-weighted) BRDF}} \underbrace{L_i(\mathbf{p}, \omega_i)}_{\text{incident lighting}} \underbrace{V(\mathbf{p}, \omega_i)}_{\text{visibility}} d\omega_i \tag{4}$$

2 路径追踪

路径追踪算法

$$L_o(p, \omega_o) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{L_i(p, \omega_i) f_r(p, \omega_i, \omega_o)(n \cdot \omega_i)}{p(\omega_i)} \tag{5}$$

```
shade(p, wo)
    Randomly choose N directions wi~pdf
    Lo=0.0
    For each wi
        Trace a ray r(p, wi)
        If ray r hit the light
            Lo += (1/N)*L_i*f_r*cosine/pdf(wi)
    Return Lo
```

3 实时阴影

(4) 可以被近似为

$$L_o(\mathbf{p}, \omega_o) \approx \frac{\int_{\Omega^+} V(\mathbf{p}, \omega_i) d\omega_i}{\int_{\Omega^+} d\omega_i} \cdot \int_{\Omega^+} L_i(\mathbf{p}, \omega_i) f_r(\mathbf{p}, \omega_i, \omega_o) \cos\theta_i d\omega_i \tag{6}$$

Shadow map

PCF (Percentage Closer Filtering) 区域取平均。

PCSS (Percentage Closer Soft Shadows) 根据接受物到遮挡物的距离决定阴影的软硬（乘个比例）。

$$w_{\text{penumbra}} = (d_{\text{receiver}} - d_{\text{blocker}}) \cdot w_{\text{light}} / d_{\text{blocker}} \tag{7}$$

1. Blocker search (getting the average blocker depth in a certain region)
2. Penumbra estimation (use the average blocker depth to determine filter size)
3. Percentage Closer Filtering

4 实时环境光

Precomputed Radiance Transfer（PRT）

$$L(\mathbf{o}) = \int_{\Omega} L(\mathbf{i}) V(\mathbf{i}) \rho(\mathbf{i}, \mathbf{o}) \max(0, \mathbf{n} \cdot \mathbf{i}) d\mathbf{i} \tag{8}$$

diffuse

$$\begin{aligned} L(\mathbf{o}) &= \rho \int_{\Omega} L(\mathbf{i}) V(\mathbf{i}) \max(0, \mathbf{n} \cdot \mathbf{i}) d\mathbf{i} \\ &\approx \rho \sum l_i \int_{\Omega} B_i(\mathbf{i}) V(\mathbf{i}) \max(0, \mathbf{n} \cdot \mathbf{i}) d\mathbf{i} \\ &\approx \rho \sum l_i T_i \end{aligned} \tag{9}$$

glossy

$$L(\mathbf{o}) \approx \sum \left(\sum l_i t_{ij} \right) B_j(\mathbf{o}) \tag{10}$$

5 实时全局光

RSM (Reflective Shadow Maps) 公式 4 改写为

$$L_o(p, \omega_o) = \int_{A_{\text{patch}}} L_i(\mathbf{q} \to \mathbf{p}) V(\mathbf{p}, \omega_i) f_r(\mathbf{p}, \mathbf{q} \to \mathbf{p}, \omega_o) \frac{\cos\theta_p \cos\theta_q}{\|q - p\|^2} dA \tag{11}$$

$$E_p(x,n) = \Phi_p \frac{\max\{0, \langle n_p | x - x_p \rangle\} \max\{0, \langle n | x_p - x \rangle\}}{\|x - x_p\|^4} \tag{12}$$

Depth, world coordinate, normal, flux

SSAO 每点发出光看比例。

$$\begin{aligned} L_o^{\text{indir}}(\mathbf{p},\omega_o) &\approx \frac{\int_{\Omega^+} V(\mathbf{p},\omega_i)\cos\theta_i d\omega_i}{\int_{\Omega^+}\cos\theta_i d\omega_i} \cdot \int_{\Omega^+} L_i^{\text{indir}}(\mathbf{p},\omega_i) f_r(\mathbf{p},\omega_i,\omega_o)\cos\theta_i d\omega_i \\ &= \frac{\int_{\Omega^+} V(\mathbf{p},\omega_i)\cos\theta_i d\omega_i}{\pi} L_i^{\text{indir}}(p) \cdot \frac{\rho}{\pi}\pi \\ &= \underbrace{k_A}_{\text{the weight-averaged visibility from all directions}} \underbrace{L_i^{\text{indir}}(p)\rho}_{\text{constant for AO}} \end{aligned} \tag{13}$$

6 实时光线追踪

Motion vector、Temporal filtering时序滤波

联合双边滤波